Chiles Mini Mu Prealgebra Round 2

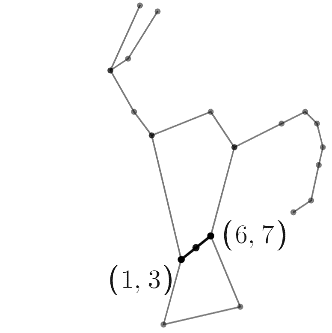
Theme: Constellations

1. Constellations are arrangements of stars that appear relatively fixed in the night sky. What is the prime factorization of the number of ways in which the letters in the word “constellation” can be arranged?

(A) (B) (C) (D) (E) NOTA

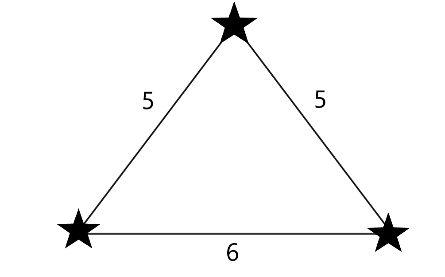
* + Answer: B
  + Solution: There are 13 letters in CONSTELLATION, but there are 2 Os, 2 Ns, 2 Ts, and 2 Ls. We must divide for overcounting, giving arrangements.

1. Orion’s Belt contains three collinear stars. Let those stars be at (3,4), (5,14), and (7,24). What is the equation of the line perpendicular to (3,4) and (7,24) that goes through (5,14)?
   * (A) (B) (C) (D) (E) NOTA
   * Answer: D
   * Solution: The line containing (3,4) and (7,24) has a slope of . The line perpendicular to it must have slope -1/5. Thus, the line is which rearranges to .
2. There is a constellation for each of the 12 zodiac signs. Find the 12th term of an arithmetic sequence with first four terms .
   * (A) 261 (B) 258 (C) -3 (D) 264 (E) NOTA
   * Answer: A
   * Solution: Because arithmetic sequences have a common difference, we can set up a system of equations: These simplify to and . Solving gives (a,b)=(6,9), so the first term is and the common difference is . Thus, the 12th term is
3. The Winter Circle is a group of six stars arranged somewhat in a circle. Find the circumference of a circle with area 5.
   * (A) (B) (C) (D) (E) NOTA
   * Answer: C
   * Solution: We know that , so Thus, the circumference is
4. I am thinking of a number. The positive difference between 11/8 of my number and 8/11 of my number is equal to 31 less than my number. What number am I thinking of? (This number happens to be the number of modern constellations recognized by the International Astronomical Union.)
   * (A) 84 (B) 85 (C) 86 (D) 87 (E) NOTA
   * Answer: E
   * Solution: The equation that represents this is . Solving for N gives 88.
5. Let the answer to the previous question be N. Find the sum of the first N odd numbers.
   * (A) 7056 (B) 7744 (C) 7225 (D) 7396 (E) NOTA
   * Answer: B
   * Solution: Since N=88, the sum of the first 88 odd numbers is 882=7744.
6. Define the star operation as x☆y = xy+x-y. Calculate (7☆2)☆(3☆5).
   * (A) 145 (B) 161 (C) 253 (D) 433 (E) NOTA
   * Answer: C
   * Solution: (7☆2)☆(3☆5) = (7\*2+7-2)☆(3\*5+3-5)=19☆13=19\*13+19-13=253
7. Pegasus is a constellation in the northern sky, named after the mythical winged horse. The body of the horse is made up of four stars arranged in a square, referred to as the Great Square of Pegasus. What is the smallest positive integer x such that is a perfect square?
   * (A) 14 (B) 21 (C) 42 (D) 84 (E) NOTA
   * Answer: C
   * Solution: The prime factorization of is . In order for this to be a perfect square, all exponents must be even, so
8. 9. A group of six friends are trying to count stars in the night sky, but it is hard to do this. They each count a different number of stars: 1, 2, 6, 15, 31, and 56. What comes next in the sequence 1, 2, 6, 15, 31, 56? (Hint: Look at the differences between consecutive terms)
   * (A) 94 (B) 141 (C) 143 (D) 92 (E) NOTA
   * Answer: D
   * Solution: The differences between consecutive terms are perfect squares. We see that , , , and so on. Thus, the 7th term is
9. The night sky seems to have an infinite number of stars. What is the value of the infinite continued fraction ?
   * (A) (B) (C) (D) (E) NOTA
   * Answer: C
   * Solution: Let Then ), giving (we can ignore the negative solution) and adding 1 from the original equation gives us the answer as sqrt2.
10. I sent a survey to the 50 people in my school’s astronomy club asking if they have ever seen the constellations Orion or Cassiopeia before. 2 did not respond. 27 of those who responded have seen Orion and 5 have seen both. How many people have only seen Cassiopeia?
    * (A) 16 (B) 18 (C) 26 (D) 28 (E) NOTA
    * Answer: C
    * Solution: Let x be the number of people who have only seen Cassiopeia. Then , which gives x=26.
11. Katharine thinks constellations are imaginary. If i=sqrt(-1), find
    * (A) -2i (B) -2 (C) 728 (D) 728i (E) NOTA
    * Answer: D
    * Solution: The powers of i repeat in a cycle of four (i,-1,-i,1). Because and , we have
12. Farhana likes constellations because she enjoys spotting patterns in the stars. The following sequence also follows a pattern: , , ... If every consecutive term from to were added together, we get a sum of 11. What is n?
    * (A) 1105 (B) 1104 (C) 1012 (D) 264 (E) NOTA
    * Answer: D
    * Solution: After rationalizing the denominators, we start to see the pattern that when we add them together, much of the terms cancel out. Adding n consecutive terms together, we notice that the only numbers left are and solving for n gives us by setting it equal to 11 gives us n = 264.
13. Farhana wants to go stargazing with Katharine to change her mind about constellations, but Katharine will only agree to go if Farhana can solve this question: How many integer solutions does |5x+12|<47 have?
    * (A) 19 (B) 18 (C) 17 (D) 16 (E) NOTA
    * Answer: B
    * Solution: We can rewrite this as -47<5x+12<47, so -59/5 < x < 7. Since we are looking for integers x, -11 <= x <= 6. There are 6-(-11)+1= 18 integer solutions.
14. Farhana answered correctly! They agree to meet at Farhana’s favorite field to stargaze at 2:36am. At 2:36, what is the measure of the larger angle formed by the minute and hour hands of a clock in degrees?
    * (A) 222 (B) 138 (C) 276 (D) 84 (E) NOTA
    * Answer: A
    * Solution: By the formula, the smaller angle is |½(60H-11M)|=|½(60\*2-11\*36)|=138. However, we want the larger angle, which is 360-138=222 degrees.
15. Katharine drives from her home to the field. She drives 15 miles in the first 1 hour but realizes she will be 30 minutes late if she continues at this speed. She doubles her speed for the rest of the way to the field and arrives 1 hour early. How many miles is the field from her home?
    * (A) 22.5 (B) 52.5 (C) 60 (D) 120 (E) NOTA
    * Answer: C
    * Solution: Let h be the number of hours from the time Katharine leaves her home to the time she must arrive at the field to be on time. Note that initially she drives at 15mph. The distance from her home to the field is both and . Setting these equal gives , so the distance is miles
16. The “ladle” of the Big Dipper is a quadrilateral with three angles measuring 105°, 100° and 70°. What is the degree measure of the fourth angle?
    * (A) 60° (B) 75° (C) 85° (D) 100° (E) NOTA
    * Answer: C
    * Solution: The angles in a quadrilateral add up to 360°. 360 - (105 + 70 +100) = 85.
17. Linsey wants to make her own constellation. If she selects 3 stars from a group of 5 stars, how many constellations can she make? (Assume that the order in which she selects them does not matter.)
    * (A) 20 (B) 25 (C) 40 (D) 50 (E) NOTA
    * Answer: E
    * Solution: The number of possible constellations = 5C3 = 10
18. James’s favorite constellation is Orion, and he wants to draw a picture of it. He draws Orion’s belt as a line segment between Alnitak at (1, 3) and Minitaka at (6, 7) on the Cartesian plane, as shown below. If Alnilam is the midpoint of this line segment, what are the coordinates of Alnilam?



* + (A) (1, 2) (B) (3, 5) (C) (1, 7) (D) (6, 3) (E) NOTA
  + Answer: E
  + Solution: The midpoint of a line segment can be found by averaging the coordinates of the endpoints.

1. While James was busy drawing Orion, Yimo stole his phone. James is at (-3, 2) on the Cartesian plane, and Yimo is at (-7, -1). How far must James travel to retrieve his phone?
   * (A) -5 (B) 2 (C) 5 (D) 10 (E) NOTA
   * Answer: C
   * Solution: Use the Pythagorean theorem to calculate distance:
2. My favorite constellation is one of the 12 zodiac constellations, and you are trying to guess it. Your guesses are random and without replacement. What is the probability that you will guess my favorite constellation on your third guess?
   * (A) 1/12 (B) 1/10 (C) 1/6 (D) 1/3 (E) NOTA
   * Answer: A
   * Solution: In order to guess my constellation on the third try, you must get the first two guesses wrong before getting it correct. Multiply the probability of each event:
3. Sarah and Karen are arguing about who can identify more constellations. Sarah says, “The number of constellations I can identify is equal to the difference between the tenth prime number and the tenth composite number.” Karen says, “The number of constellations I can identify is equal to + . Who can identify more constellations, and how many can that person identify?
   * (A) Sarah; 11 (B) Sarah; 15 (C) Karen; 10 (D) Karen; 14 (E) NOTA
   * Answer: D
   * The tenth prime number is 29 and the tenth composite number is 18, so Sarah can identify 11 constellations. The cube root of 729 is 9 and the square root of 25 is 5, so Karen can identify 14 constellations
4. Farhana sees a constellation with three stars, as shown below. The three stars form an isosceles triangle, with two sides of length 5 units and one side with length 6 units. What is the total area of the triangle?



* + (A) 6 units2 (B) 10 units2 (C) 12 units2 (D) 24 units2 (E) NOTA
  + Answer: C
  + Solution: We can divide this triangle into two right triangles with side lengths 3, 4 and 5 units. The area of one of these triangles is 6, so the area of the larger triangle is 12.

1. Mari’s favorite constellation is Gemini. The number of stars in the Gemini constellation is equal to the sum of the prime factors of 220. How many stars are in Gemini?
   * (A) 14 (B) 18 (C) 20 (D) 22 (E) NOTA
   * Answer: B
   * Solution: 220 = 22\*5\*11, so the prime factors are 2, 5, and 11
2. Nonoko wants to use Katharine’s binoculars to look at her favorite constellation. Katharine will only give up the binoculars if Nonoko can tell her the 11th prime number. What is the correct answer?
   * (A) 21 (B) 23 (C) 29 (D) 31 (E) NOTA
   * Answer: D
   * Solution: The first 11 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31.
3. Lilly wants to travel from Betelgeuse to Bellatrix (both of which are stars in the Orion constellation) and stop at Orion’s belt on the way. Given that Betelgeuse is at (1, 7), Bellatrix is at (6, 5), and Orion’s belt is the line y = 0, determine the minimum distance Lilly needs to travel.
   * (A) 5 (B) 8 (C) 10 (D) 13 (E) NOTA
   * Answer: D
   * Solution: In order to find the minimum distance, reflect Bellatrix over y = 0 to get the point (6, -5). Find the distance between this point and Betelgeuse using the Pythagorean theorem:
4. Linda’s favorite star is the Sun because it's only 93 million miles away. If 1 mile = 1.6 kilometers, how many kilometers is the Sun from the Earth? Round your answer to the nearest ten.
   * (A) 130 million (B) 140 million (C) 150 million (D) 160 million (E) NOTA
   * Answer: C
   * Solution: 93 million \* 1.6 = 148.8 million
5. If there are 88 major constellations in the night sky, and 12 of those are zodiac constellations, what is the probability that a randomly selected constellation will NOT be a zodiac constellation?
   * (A) 3/88 (B) 3/22 (C) 2/11 (D) 19/22 (E) NOTA
   * Answer: D
   * Solution: If there are 12 zodiac constellations, there must be 76 constellations that aren’t zodiacs. 76/88 = 19/22
6. Farhana wants to know what the most popular constellation is, so she asks each of her friends. 7 of her friends say Orion is their favorite, 4 say the Big Dipper, 3 say Leo, and 1 says Cassiopeia. What percentage of her friends like Leo the most?
   * (A) 10% (B) 15% (C) 20% (D) 25% (E) NOTA
   * Answer: C
   * Solution: (3 people who liked Leo) / (15 total people) = 0.2 or 20%
7. Linsey is putting star stickers on her wall to make a constellation with 4 stars in a straight line. Linsey has small, medium, and large stickers. How many options does Linsey have to make this constellation?
   * (A) 4 (B) 64 (C) 81 (D) 1000 (E) NOTA
   * Answer: C
   * Solution: Each star can be one of three different sizes, and there are 4 stars total so Linsey has 34 = 81 options